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Branes, waves and AdS orbifolds

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Abstract: The embedding of a Taub-NUT space in the directions transverse to the worldvolume of branes describes branes at (spherical) orbifold singularities. Similarly, the embedding of a pp-wave in the brane worldvolume yields an AdS orbifold. In case of the D1-D5-brane system, the AdS_3 orbifolds yields a BTZ black hole; as we will show, the same holds for D3-branes corresponding to AdS_5. In addition we will show that the AdS orbifolds and the spherical orbifolds are U-dual to each other. However in contrast to spherical orbifolds the AdS orbifolds lead to a running coupling, which is in the IR inverse to the coupling of the spherical orbifold. A discussion of the general pp-wave solution in AdS space is added.

Keywords: Superstrings and Heterotic Strings, String Duality, Black Holes in String Theory, Supersymmetry Breaking.

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Contents

1. Introduction

Besides flat spacetime, the near horizon geometry of regular branes, given by a direct product of an anti de Sitter (AdS) space and a sphere, is maximal supersymmetric. By projecting out a discrete subgroup of the isometry group the supersymmetry will be partially broken. Due to the direct product of spacetime, there are two distinct possibilities

$$AdS_p \times S_q/\Gamma\,, \qquad AdS_p/\Gamma \times S_q\,. \tag{1.1}$$

The first class of orbifolds, called spherical orbifolds, are well known from string compactification and have been discussed in the context of AdS/CFT correspondence [1, 2, 3] in [4, 5, 6]. On the other hand a well-known example for an AdS orbifold is the BTZ-black hole which corresponds to AdS_3/Z_k [7], see also [8]. While spherical orbifolds are related to an embedding of a Taub-NUT (TN) space in the transversal space, we will show that AdS orbifolds are obtained by (non-standard) pp-wave embeddings in the brane world volume directions. Since the anti-de Sitter space is defined as a hyperboloid in a space with two time-like coordinates, one can embed up to two waves, i.e., one can consider up to two independent orbifolds, which is also true for spherical orbifolds. These double orbifolds correspond to an embedding of intersections of supergravity solutions, e.g., $AdS_5 \times S_5/(Z_k \times Z_{k'})$ is obtained in the decoupling limit from the intersection: D3 $\times$ TN $\times$ TN (see, e.g., [9, 10]) and as we will argue $AdS_5/(Z_k \times Z_{k'}) \times S_5$ corresponds to the intersection: D3 $\times$ D$(-1)\times$ wave. Notice, from the F-theory perspective the D(-1)-brane (D-instanton) is an "internal" wave in the two hidden dimensions [11].

Obviously, from the supergravity point of view one can dualize the wave into a Taub-NUT space and vice versa. So, both orbifolds should be equivalent. In fact we will show that the AdS orbifolds and the spherical orbifolds are U-dual to each other. But, we have to keep in mind, that both orbifolds acts very differently: the spherical orbifold breaks the R-symmetry, whereas for the AdS-orbifolds the Lorentz symmetry is broken (see discussion). The gauge theories, which correspond to the spherical orbifolds, have reduced number of supersymmetries (compared to $N = 4$ in four dimensions), but are still conformal field theories with vanishing β-function. On the other hand, the AdS-orbifolds yield a running coupling from the ultraviolett (UV) to the infrared (IR). This issue will be discussed for AdS_3 in the next section and for AdS_5 in section 3. Various aspects of supergravity solutions with broken supersymmetries and possibly running couplings in the corresponding gauge theories, but with unbroken Lorentz symmetry, were recently discussed in [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24].

2. Three-dimensional orbifolds

The 3-dimensional orbifolds are the simplest cases and the natural framework for their discussion is the D1-D5-brane system (or also of the F1-NS5), with the metric given by

$$ds^2 = \frac{1}{\sqrt{H_1 H_5}}\big(- dt^2 + dy^2\big) + \sqrt{H_1 H_5}\,\big(dr^2 + r^2\, d\Omega_3\big) + ds^2_{\rm int}\,. \tag{2.1}$$

For our purpose here, we can identify the two harmonics $H_1 = H_5 = H = 1 + Q/r^2$ and get a self-dual string in 6 dimensions. In the decoupling limit (equivalent to the near-horizon limit with large charges) the internal 4-d metric $ds^2_{\rm int}$ becomes a flat euclidean space and the remaining 6-d space becomes $AdS_3 \times S_3$

$$ds^2 = \left[\frac{r^2}{l^2}\big(- dt^2 + dy^2\big) + l^2\,\frac{dr^2}{r^2}\right] + l^2\, d\Omega_3\,, \quad l^2 = \sqrt{q_1 q_5} = Q \sim N\,, \tag{2.2}$$

where l is the S_3 radius which scales with the number of self-dual strings N.

We start with the discussion of the S_3-orbifold (S_3/Z_k) and in order to fix our notation and for later convenience let us repeat some known facts, see, e.g., [25, 26]. An S_3 with a radius l is defined by

$$l^2 = \big(X^0\big)^2 + \big(X^1\big)^2 + \big(X^2\big)^2 + \big(X^3\big)^2 = z_1\bar{z}_1 + z_2\bar{z}_2\,, \tag{2.3}$$

where we introduced the complex coordinates

$$z_1 = l\, e^{i\,(\theta_R+\theta_L)/2}\,\cos\frac{\lambda}{2}\,, \qquad z_2 = l\, e^{i\,(\theta_R-\theta_L)/2}\,\sin\frac{\lambda}{2}\,. \tag{2.4}$$

Inserting these coordinates, the S_3 metric becomes

$$ds^2 = dz_1\, d\bar{z}_1 + dz_2\, d\bar{z}_2 = \frac{l^2}{4}\Big(d\lambda^2 + d\theta_R^2 + d\theta_L^2 + 2\cos\lambda\, d\theta_R\, d\theta_L\Big) = l^2\, d\Omega_3\,, \tag{2.5}$$

where $\lambda \in (0,\pi); \theta_R \in (0,2\pi);\ \theta_L \in (0,4\pi)$. As next step we consider the orbifold given by the lens space S_3/Z_k

$$z_1 \simeq e^{2\pi i/k} z_1\,, \qquad z_2 \simeq e^{-2\pi i/k} z_2 \tag{2.6}$$

which corresponds to the identification

$$\theta_L \simeq \theta_L + \frac{4\pi}{k}\,. \tag{2.7}$$

Applying this identification to a given supergravity solution is equivalent to an embedding of a Taub-NUT-space with NUT charge p ($p \sim k$), i.e.,

$$\begin{aligned} dr^2 + r^2\, d\Omega_3 \longrightarrow & \frac{1}{1+(p/r)}\Big[dz + p(\pm 1 + \cos\lambda)\, d\phi\Big]^2 + \\ & + \left(1+\frac{p}{r}\right)\Big[dr^2 + r^2\big(d\lambda^2 + \sin^2\lambda\, d\theta_R^2\big)\Big]\,. \end{aligned} \tag{2.8}$$

Making this replacement in (2.1) and taking for the harmonic functions $H_1 = H_5 = 1 + (Q/r)$ and $z = p\,\theta_L$ we get in the decoupling limit (or near-horizon limit) exactly the same S_3 metric as before, but the consistency of a Taub-NUT space requires the periodic identification (2.7).

What is the analogous orbifold for the AdS_3 space? As for the sphere, also AdS_3 is defined as an hyperboloid, but now with two time-like directions

$$-l^2 = -\big(X^0\big)^2 + \big(X^1\big)^2 + \big(X^2\big)^2 - \big(X^3\big)^2 = -z_1^+ z_1^- + z_2^+ z_2^- \tag{2.9}$$

with the four real coordinates given by

$$z_1^{\pm} = l\, e^{\pm(\theta_R+\theta_L)/2}\, \cosh\frac{\lambda}{2}\,, \qquad z_2^{\pm} = l\, e^{\pm(\theta_R-\theta_L)/2}\, \sinh\frac{\lambda}{2}\,. \tag{2.10}$$

The metric becomes now

$$ds^2 = -dz_1^+\, dz_1^- + dz_2^+\, dz_2^- = \frac{l^2}{4}\Big(d\lambda^2 + d\theta_R^2 + d\theta_L^2 + 2\cosh\lambda\, d\theta_R\, d\theta_L\Big)\,, \tag{2.11}$$

which coincides with the AdS_3 metric given in (2.2) if $e^\lambda = r^2 \gg 1$ and $\theta_{R/L} = (2/l^2)(y \pm t)$. In analogy to the S_3 case the AdS_3 orbifold is given by the identification

$$z_1^{\pm} \simeq e^{\pm 2\pi/k} z_1^{\pm}\,, \qquad z_2^{\pm} \simeq e^{\mp 2\pi/k} z_2^{\pm} \tag{2.12}$$

which is equivalent to

$$\theta_L \simeq \theta_L + \frac{4\pi}{k}\,. \tag{2.13}$$

Notice, there is a subtle difference between the spherical and the AdS orbifolds. For the spherical case the orbifold acts on a compact direction and repeating the orbifold

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k times becomes trivial; one returns to the starting point. On the other hand, in the AdS case the orbifold acts along a non-compact lightcone direction and thus one does not return after k steps. But nevertheless, the transformation (2.12) represents a discrete symmetry of the defining equation (2.9) which is parameterized by the integer k and which we will also denote by Z_k.

Like the S_3/Z_k orbifold is obtained by an embedding of a Taub-NUT space in the spherical part of the solution, also the AdS_3/Z_k orbifold corresponds to an embedding of a supergravity solution, but now it is a pp-wave in the worldvolume part. In fact, as it has been widely discussed in matrix theory [27, 28], (extremal) waves yields a discretization of a lightcone direction corresponding to a quantized momentum number. A wave embedding corresponds to the replacement

$$\begin{aligned} -dt^2 + dy^2 \longrightarrow & -dt^2 + dy^2 + (H-1)(dy - dt)^2 = \\ & = \frac{2}{S_+ + S_-}(dy + S_+\, dt)(dy - S_-\, dt) \end{aligned} \tag{2.14}$$

with the harmonic function $H = 1 + (r_0/r)^2$ and $S_\pm = H^{-1} \pm (H^{-1} - 1)$ $(r_0 \sim k)$. After this replacement and a shift in the radial coordinate $(r^2 \to r^2 - r_0^2)$ the AdS_3 part in (2.2) becomes

$$ds^2 = -\left(\frac{r^2 - r_0^2}{rl}\right)^2 dt^2 + \left(\frac{rl}{r^2 - r_0^2}\right)^2 dr^2 + \left(\frac{r}{l}\right)^2 \left(dy - \frac{r_0^2}{r^2}\, dt\right)^2 . \tag{2.15}$$

This is the (extreme) 3-d BTZ black hole which in fact represents an AdS_3 orbifold [7]. It is locally equivalent to (2.11) and globally it is well defined as long as the lightcone direction $y - t = \theta_L$ in (2.14) is periodically identified.

Therefore, one has the dictionary

$$\begin{aligned} \mathrm{D1} \times \mathrm{D5} + \mathrm{TN} &\simeq (AdS_3) \times (S_3/Z_k)\ , \\ \mathrm{D1} \times \mathrm{D5} + \mathrm{wave} &\simeq (AdS_3/Z_k) \times (S_3)\ . \end{aligned} \tag{2.16}$$

Of course both configurations are dual to each other, e.g., stretching the branes along the following directions (the NUT and wave directions are indicated by $\otimes$ and o resp.)

$$\begin{array}{cccccc} \hline 1 & 2 & 3 & 4 & 5 & 6 \\ \hline \times & \times & \times & \times & \times & \\ \times & & & & & \\ & & & & & \otimes \\ \hline \end{array} \quad \longleftarrow \mathrm{U} \longrightarrow \quad \begin{array}{cccccc} \hline 1 & 2 & 3 & 4 & 5 & 6 \\ \hline \times & \times & \times & \times & \times & \\ \times & & & & & \\ o & & & & & \\ \hline \end{array} \tag{2.17}$$

the U-duality group element, that transforms both configurations into one another is given by a sequence of T and S dualities

$$\mathrm{U} = \mathrm{T}_{1236}\, \mathrm{ST}_{2345}\, \mathrm{ST}_{1456}\ . \tag{2.18}$$

Notice, although both orbifolds are duality equivalent, they act very differently. From the worldvolume point of view, the spherical orbifold "operates" in the internal space and leaves the external spacetime invariant. On the other hand the AdS orbifold involves the time or a lightcone direction and corresponds in the decoupling limit to an infinite momentum frame, as discussed in matrix theory. In addition, the periodicity introduced by the orbifold corresponds to a compact direction with a non-constant radius $R = R(r)$, where r is the radial coordinate of the AdS space. Effectively this means, that in contrast to a spherical orbifold, an AdS orbifolds yields a running couplings.

Let us discuss this point in more detail. Orbifolds change only the global structure (identifications along a compact direction), locally the spaces are unchanged, i.e., they are still S_3 or AdS_3. Especially the scalars like the dilaton are not affected by this procedure, at least not from the supergravity point of view. However, since the non-trivial global structure corresponds to a compact coordinate the situation at hand is physically equivalent to the case where we T-dualize this compact direction. By this T-duality the dilaton will get a dependence on the radius of the compact direction, i.e., the non-trivial global structure becomes "visible". And since the radius of the compact coordinate is not constant, the dilaton will run from the UV related to $R(r = \infty)$ towards the IR corresponding to $R(r = r_0)$, where the IR point $r = r_0$ is defined as the maximal possible extension while keeping the coordinate system[1] at $r = \infty$. Therefore, the IR appears either as further boundary of spacetime or as a horizon, see also [30]. To be conrete, let us T-dualize the compact y direction. We obtain from (2.15)

$$ds^2 = \frac{1}{H}\left[du\,dv + \frac{r_0^4/l^2}{r^2}\,du^2\right] + \frac{l^2}{r^2}\,dr^2\,, \quad e^{-2\phi} = \frac{r^2}{l^2}H\,, \quad B = \frac{1}{2H}\,du\wedge dv \tag{2.19}$$

with $H = 1 + (r_0/r)^2$, and we shifted the horizon from $r = r_0$ in (2.15) to $r = 0$ by $r^2 \to r^2 + r_0^2$. Asymptotically, this metric becomes flat and the dilaton linear

$$ds^2 = -dt^2 + dy^2 + l^2\,d\lambda^2\,, \qquad \phi = \lambda \tag{2.20}$$

($l/r = e^{\lambda}$). Therefore, the dilaton coupling flows from the UV-free situation towards a non-trivial IR fixpoint near the horizon

$$e^{2\phi} = \begin{cases} 0\,, & r\to\infty \quad \text{(UV region)}\,, \\ \dfrac{l^2}{r_0^2}\,, & r = 0 \quad \text{(IR region)} \end{cases} \tag{2.21}$$

or if we express the dilaton by dimensionless (integer-valued) quantities, we find in the IR region

$$e^{2\phi_{\rm IR}} \sim \frac{N}{k}\,, \tag{2.22}$$

[1] Changing the coordinate system corresponds in the field theory to an operator reparameterization, which would imply a different RG behavior [29].

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where k was the momentum number related to the orbifold and N is the number of self-dual strings.

Let us also compare this result with the S_3 orbifold, where we have to embed the Taub-NUT space in the transversal part. Because the NUT direction represents an isometry direction, the D1-D5-brane system is localized only in 3 transversal coordinates and therefore we have to replace the harmonic functions by $H_1 = H_5 = 1 + Q/r$. Hence, making the replacement (2.8) in (2.1) we find in the decoupling limit where $Q \sim N$ is large

$$\begin{aligned} ds^2 =& \frac{r}{l^2}\big(- dt^2 + dy^2\big) + \frac{Q}{r}\Big(1 + \frac{p}{r}\Big)\, dr^2 + \\ &+ \frac{Q}{r}\,\frac{1}{1+p/r}\Big[dz + p(\pm 1 + \cos\lambda)\, d\phi\Big]^2 + \\ &+ Q(r+p)\Big(d\lambda^2 + \sin^2\lambda\, d\theta_R^2\Big)\,. \end{aligned} \tag{2.23}$$

If in addition p is large the first line becomes the AdS_3 space and the second line represents the S_3 orbifold. As for the AdS_3 orbifold we can read off an effective coupling from the compact z direction, e.g., by employing T-duality. But this time we do not see a flow behavior for the dilaton. The orbifold itself describes the field theory in the IR region and the constant dilaton reads

$$e^{2\phi_{\rm IR}} = \frac{p}{Q} \sim \frac{k}{N}\,, \tag{2.24}$$

where k is the integer parameterizing the NUT charge p, which is related to the Z_k orbifold ($p \sim k$). Comparing this expression with (2.22), we see that the coupling constant has been inverted. The strong-weak coupling duality between both orbifolds reflects the fact that upon compactification the wave and Taub-NUT spaces are S-dual to each other.

3. Orbifolds of AdS_5

We will start with the discussion of a single orbifold AdS_5/Z_k and later we will comment on the double orbifold $AdS_5/(Z_k \times Z_{k'})$. In comparison to AdS_3, the AdS_5 space has two additional euclidean coordinates, i.e., it is defined by a hyperboloid in a 6-d space with two timelike directions

$$\begin{aligned} -l^2 &= -\big(X^0\big)^2 + \big(X^1\big)^2 + \big(X^2\big)^2 - \big(X^3\big)^2 + \big(X^4\big)^2 + \big(X^5\big)^2 = \\ &= -z_1^+ z_1^- + z_2^+ z_2^- + w\bar{w} \end{aligned} \tag{3.1}$$

with

$$\begin{aligned} z_1^\pm &\equiv X_0 \pm X_1 = \sqrt{R^2 + l^2}\, e^{\pm\psi_1}\,, \\ z_2^\pm &\equiv X_2 \pm X_3 = R\cos\theta\, e^{\pm\psi_2}\,, \\ w &\equiv X_4 + iX_5 = R\sin\theta\, e^{i\chi}\,. \end{aligned} \tag{3.2}$$

The orbifold action on the AdS_5 space is defined as before, i.e., via the embedded AdS_3 space, see eq. (2.12). Obviously at $\theta = 0$ we get back the AdS_3 space as given in (2.9) with $\psi_{1/2} = (\theta_R \pm \theta_L)/2$. In these coordinates the metric becomes

$$ds^2 = -R^2 \cos^2\theta \, d\psi_2^2 + \left(R^2 + l^2\right) d\psi_1^2 + \frac{l^2}{R^2 + l^2} \, dR^2 + R^2 \left(d\theta^2 + \sin^2\theta \, d\chi^2\right) . \quad (3.3)$$

Or, after introducing

$$\psi_1 = -\frac{r_+}{l^2} \, y \,, \qquad \psi_2 = \frac{r_+^2 - r_-^2}{r_+} \, t + \frac{r_-}{l^2} \, y \quad (3.4)$$

we find

$$\begin{aligned} ds^2 = & \cos^2\theta \left[-\frac{(r^2 - r_-^2)(r^2 - r_+^2)}{l^2 r^2} \, dt^2 + \frac{r^2}{l^2} \left(dy - \frac{r_-}{r_+} \left(1 - \frac{r_+^2}{r^2}\right) dt \right)^2 \right] + \\ & + \frac{l^2 r^2}{(r^2 - r_-^2)(r^2 - r_+^2)} \, dr^2 + l^2 \, \frac{r^2 - r_+^2}{r_+^2 - r_-^2} \left(d\theta^2 + \sin^2\theta \, d\chi^2\right) + \\ & + \frac{r_+^2}{l^2} \, \frac{r^2 - r_-^2}{r_+^2 - r_-^2} \, \sin^2\theta \, dy^2 \, . \end{aligned} \quad (3.5)$$

This solution is one example of topological AdS black holes [31] (see also [32]), which are locally equivalent to the AdS space, but globally different. In fact, for $\theta = 0$ it becomes the BTZ black hole with the two horizons at $r = r_\pm$. It is not only a solution of 5-d Einstein — anti de Sitter theory, but it solves also 5-d Chern-Simons with SO$(4,2)$ gauge group; which includes the Gauss-Bonnet term.

In the extreme limit both horizons coincides and to make the limit regular, one has also to rescale θ in a way that

$$\theta \longrightarrow \frac{\sqrt{r_+^2 - r_-^2}}{l^2} \, \sigma \,, \quad r_+^2 \longrightarrow r_-^2 = r_0^2 \text{ with } \sigma \text{ fix} \, . \quad (3.6)$$

After replacing $r^2 - r_0^2 \to r^2$, $2t = v$, $y = u$ we obtain a 3-brane with a pp-wave

$$\begin{aligned} ds^2 &= \frac{r^2}{l^2} \Big[- dv \, du + H \, du^2 + d\sigma^2 + \sigma^2 \, d\chi^2 \Big] + l^2 \, \frac{dr^2}{r^2} \,, \\ H &= 1 + \frac{r_0^2}{l^4} \, \sigma^2 + \frac{r_0^2}{r^2} \end{aligned} \quad (3.7)$$

but due to the σ dependence of H it is *not* the standard wave.

To understand this solution better let us discsuss the general wave solution in an AdS space, see also [33]. The standard wave ansatz in an AdS_{p+2} space reads $(z = (l^2/r))$

$$ds^2 = \frac{l^2}{z^2} \Big[- dv \, du + H(u, x_i, z) \, du^2 + dx_1^2 + \cdots + dx_{p-2}^2 + dz^2 \Big] . \quad (3.8)$$

This ansatz is a solution of AdS_{p+2} gravity ($R_\mu^\nu = -((p+1)/l^2)\delta_\mu^\nu$) if H solves the laplacean equation

$$\Delta H \equiv \frac{1}{\sqrt{g}}\partial_\mu\big(\sqrt{g}\,g^{\mu\nu}\partial_\nu H\big) = \frac{z^2}{l^2}\Big(\partial_\|^2 + z^p\partial_z\,z^{-p}\partial_z\Big)H(u,\vec{x},z) = 0\,, \tag{3.9}$$

where $\vec{x} = (x_1, x_2, \dots)$ and, as typical for pp-waves, one can allow for a general u dependence; for a recent discussion of this u dependence in the AdS/CFT correspondence see, e.g., [34]. Let us give some special cases

(i) if $H = H(z)$: the solution is $H = h + cz^{p+1}$, which corresponds to the expected harmonic function with respect to the $9-p$ transversal brane directions (note $z \sim 1/r$).

(ii) if $H = H(\vec{x})$: one gets an harmonic function with respect to the worldvolume coordinates, e.g., a logarithm for the 3-brane.

(iii) if $H = f(\vec{x})g(z)$: the solution becomes $H \sim e^{\pm i\vec{p}\cdot\vec{x}}\,z^\nu \mathcal{K}_\nu(|p|z)$, $\nu = (1+p)/2$ where $\mathcal{K}_\nu(|p|z)$ is the modified Bessel functions. At the asymptotic boundary ($z = 0$, UV region) $z^\nu\mathcal{K}_\nu \sim$ constant and H parameterize a plane wave whereas in the IR ($z \to \infty$) H vanishes.

(iv) finally if $H = H(|\vec{x}|^2 + z^2)$: the solution reads

$$H = h + c\big(|\vec{x}|^2 + z^2\big)\,. \tag{3.10}$$

For AdS_5 this last solution coincides exactly with solution obtained in (3.7) if $|\vec{x}| = \sigma$; $z = l^2/r$ and $c = r_0^2/l^4$. Notice, that this regular solution is the same exact wave solution which is also known from the flat space case. Moreover, this wave breaks explicitly the worldvolume isometries; only the lightcone direction u, v represents still isometries.

By a simple shift in v we can absorb h and in order to avoid a conical singularity at $\sigma = z = 0$ we have to do a periodical identification along the lightcone direction u, which corresponds exactly to our orbifold. This was also the case for AdS_3: for the non-extreme case the identification is along a spatial direction whereas in the extreme case a lightcone direction is periodically identified. Moreover, this compact direction breaks the scale invariance and since its radius is not constant, it corresponds to a running coupling. Again reading off the dilaton from this compact direction we find $e^{-2\phi} = g_{uu} = (r^2/l^2)H$ or

$$e^{2\phi} = \begin{cases} 0\,, & r \to \infty \quad \text{(UV region)}\,, \\ \dfrac{l^2}{R_0^2} \sim \dfrac{N}{k}\,, & r = 0 \quad \text{(IR region)}\,. \end{cases} \tag{3.11}$$

Notice, that the σ-dependence drops out!

We can also compare this result with the S_5 orbifold, related to a Taub-NUT embedding into the S_5. Reading off the dilaton coupling from the radius of the NUT direction, we find

$$e^{2\phi_{\rm IR}} \sim \frac{k}{N}\,, \tag{3.12}$$

where the radius of S_5 scales with N and k corresponds to the NUT charge which corresponds to the orbifold. Recall for this spherical orbifold we do not get a running coupling, but only the effective value in the infrared (the compact NUT direction introduces a scale), which is again opposite to the value from the AdS orbifold. Both orbifolds should be again dual to each other. But note, the naive U-duality that relates the 3+wave configuration to 3+Taub-NUT includes T-duality transformations along the worldvolume directions of the 3-brane. In the case at hand however, the wave (3.10) depends explicitly on these coordinates and therefore the standard T-duality rules do not apply. It would be interesting to work out in which sense the supergravity solutions of both orbifolds are dual to each other.

Finally, let us also mention, that because AdS_5 is defined as a hyperboloid in the a 6-d space with *two* time-like direction, this space can accommodate exactly two independent waves (a wave requires always a time-like direction). This suggest, that like S_5 also AdS_5 allow for at most two independent orbifold actions. However by reducing the 6-d space to AdS_5 we loose one time and therefore the second wave cannot show up as a second wave in AdS_5. On the other hand in the F-theory approach to type IIB string theory, the D-instanton corresponds also to a wave with respect to the "hidden time" [11]. Hence, the corresponding supergravity solution should include a wave as well as a D-instanton and it is straightforward to construct this solution. In the Einstein frame the IIB dilaton $e^{-\phi}$ and RR scalar l ($S^{\pm} = l \pm e^{-\phi}$) solve the equations of motion if (see, e.g., [35])

$$S^+ = \text{constant}\,, \qquad S^- = \frac{1}{\bar{H}}\,, \tag{3.13}$$

where $\bar{H}$ is also a solution of the laplacean equation (3.9). Thus in the string frame we obtain the metric

$$ds^2 = \sqrt{\bar{H}}\,\frac{l^2}{z^2}\Big[- dv\,du + H(u, x_i, z)\,du^2 + dx_1^2 + dx_2^2 + dz^2\Big]\,. \tag{3.14}$$

The harmonic function $\bar{H}$ could be any of the cases discussed after (3.9). For further discussions of D-instantons in the AdS/CFT correspondence we refer to [36, 37, 38]. We expect, that the wave as well as the D-instanton break 1/2 of supersymmetry so that in total only 1/4 of supersymmetry will be preserved and we will leave it for future investigation to work out the precise action of the $Z_k \times Z_{k'}$ orbifold on the AdS_5 space as well as to work out the concrete U-duality element that maps the AdS_5 orbifold onto the $S_5/(Z_k \times Z_{k'})$ orbifold.

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4. Discussion

In the near-horizon limit, regular branes factorize into an anti de Sitter space and a sphere yielding two distinct possibilities for orbifolds: $AdS \times S/\Gamma$ or $AdS/\Gamma \times S$. In this paper we focused on AdS-orbifolds and compared them with spherical orbifolds. Both cases correspond to an embedding of a supergravity solution: a Taub-NUT space for spherical orbifolds and pp-waves for the AdS-orbifolds. Both the supergravity solutions acts quite differently, the Taub-NUT space breaks the R-symmetry related to rotations in the transverse space and the wave breaks the worldvolume rotational symmetry. From the field theory point of view the R-symmetry is internal, whereas the worldvolume rotations broken by pp-waves are part of the Lorentz group. Notice, pp-waves are interpreted as gravitons with a given momentum and in the decoupling limit this momentum should be large.

In projecting out the discrete subgroup, one has to truncate the spectrum onto an invariant subsector. For both orbifolds one has to project out one chirality: for spherical orbifolds it is one chirality with respect to the NUT direction, as discussed in [25] for the 3-d case; and for the AdS-orbifold it is one chirality with respect to the momentum modes corresponding to the wave. For the AdS_3 case the corresponding states are the chiral primaries corresponding to momentum modes travelling only in one direction along the (extremal) string, see [39].

Finally, let us stress that the pp-wave yielding an AdS-orbifold is in general not the "standard" pp-wave that corresponds to a harmonic function. Instead, it is the exact wave solution that quadratically increases in the transversal space, see (3.10), and the periodic identification, which avoids a conical singularity, represents the orbifold. So, this solution breaks the translational invariance along the brane worldvolume (localized wave). Only the wave direction represents still an isometry direction and by T-dualizing this direction one can construct brane intersection which are localized and supersymmetric.

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